# Assignment 2 – COMP2270

* 1. True;   
     For a finite language *A* and another language *B*,   
     where |*L1*| = {*n1* : *n1* is a finite number and *B* ⊆ *A*},   
     ∀B = |*B*|= {*n2* : *n2* ≤ *n1*}
  2. True;   
     For a regular language *L1*, where *L1* is composed of strings ***s1, s2, … sn*** or regular expression ***s1* ⋃ *s2* ⋃ *…* ⋃ *sn***, every possible subset has an equivalent regular expression.
  3. False;  
     Proof by counter-example:

Where *B* = {*w* ∈{*a*, *b*}\* : *w* has an *a* followed by zero or more *b*} and

*A* = {*w* ∈{*a*, *b*}\* : *w* has an optional *a* is followed by zero or more a *b*}

Each string in language *A* is a possible string in language *B*.

However, *B* is regular and *A* is not, as shown in *Fig. 1.1* and *Fig. 1.2.*



*Fig. 1.1: DFSM for language B*

*Fig. 1.2: NDFSM for language A*

* 1. False;
  2. The context-free grammar *G* for language *L* = {*aibk* : *k* = 4*i* + 2 and *i, k* ≥ 0} is:

*G* = ({*S, a, b*}, {*a, b*}, *R*, *S*)

where *R* = {

*S* → *aSbbbb* | *bb* }

* 1. The context-free grammar *G* for language *L* = {*anbp* : *p* ≥ *n*, *p* – *n* is odd} is:

*G* = ({*S, T, a, b*}, {*a, b*}, *R*, *S*)

where *R* = {

*S* → *Tb* | *Sbb*

*T* → *aS* | ε}

* 1. The bear shot Fluffy with the rifle: 

*Fig. 3.2: unlikely parse tree for Q3 a)*

*Fig. 3.1: likely parse tree for Q3 a)*

The likely context is that the bear used the rifle to shoot Fluffy (*Fig. 3.1*). The less-likely interpretation is that the bear shot Fluffy while Fluffy had the rifle (*Fig. 3.2*). Hence, the most probable parse tree is ***Fig. 3.1***.

* 1. Fluffy likes the girl with the chocolate:



*Fig. 3.4: unlikely parse tree for Q3 b)*

*Fig. 3.3: likely parse tree for Q3 b)*

The likely context is that Fluffy likes the girl who is in possession of the chocolate (*Fig. 3.3*). The less-likely interpretation is that Fluffy uses the chocolate to like the girl (*Fig. 3.4*). Hence, the most probable parse tree is ***Fig. 3.3***.

* 1. The leftmost derivation of string **ab#cc** is:

*S* → *T#T* → *ABA#T* → **a** *ABA#T* → **a** *BA#T* → **ab** *BA#T* → **ab** *A#T* → **ab** *#T* → **ab#** *T* → **ab#** C → **ab#c** C → **ab#cc**

* 1. *G* can be proven ambiguous by showing that at least one string it produces is ambiguous. The string **a#cc** is ambiguous – two possible parse trees for this string are presented in *Fig 4.1* and *Fig 4.2*:

 

*Fig. 4.2: second parse tree for* ***a#cc***

*Fig. 4.1: first parse tree for* ***a#cc***

* 1. *L* = {a*i*b*k* : *k* = 3*i* + 3}  
     *M* = ({1, 2, 3}, {a, b}, {a}, Δ, 1, {3}), where  
     Δ = {  
     ((1, a, ε), (1, a)),   
     ((1, ε, ε), (2, ε)),   
     ((2, b, aaa), (2, ε)),   
     ((2, ε, aaa), (3, ε)) }



*Fig. 5.1: FSM for   
L = {aibk : k = 3i + 3}*

* 1. {a*i*b*j*c*k* , *i* > *k*, 0 ≤ *j* < 3, *k* ≥ 0}  
     *M* = ({1, 2, 3, 4, 5, 6}, {a, b, c}, {a, b}, Δ, 1, {6}), where  
     Δ = {  
     ((1, a, ε), (1, a)),   
     ((1, a, ε), (2, a)),   
     ((2, b, ε), (3, b)),   
     ((2, ε, ε), (5, ε)),  
     ((3, b, ε), (4, b)),  
     ((3, ε, b), (5, ε)),  
     ((4, ε, bb), (5, ε)),  
     ((5, ε, a), (6, ε)),  
     ((6, c, a), (6, ε)) }

*Fig. 5.2: PDA for   
L = {aibk : k = 3i + 3}*

* 1. *aa*, *bb*, *aaaa*, *abba*
  2. *G* = ({*S, T, a, b*}, {*a, b*}, R, S), where *R* =

{ *S* → *aSa* | *aTa* | *bTb*

*T* → *bTb* | ε }

* 1. 

*Fig. 6.1: PDA for   
L = L1 ∩ L2*, *where  
L1 = {wwR : w ∈ {a,b}\*},  
L2 = {anb\*an: n≥0}*

* 1. *L* is not regular;   
     With Pumping Theorem and proof by construction,   
     let *L* = {*w* ∈ {*an(b* ∪ *b)\*an* } : *n* ≥ 0, |*w*| ≥ 2} where *L* = *L1**∩ L2*, and *w* = *akbkbkak*.   
     Also let *w* = *xyz*, |*xy*| ≤ *k*, *y* ≠ ε and *y* = {*ap* : *p* > 0 } and *y* ∈ (*aa*)+.   
     Where *w*q=0 = *ak-pbkbkak*, *w* ∉ *L*, therefore *L* is not regular.
  2. True; if the Kleene plus of a language is context-free, then it is trivially obvious that the language itself is context-free. In other words, if *L1∪ L2 ∪ … ∪ Ln* is context-free, than all elements of *L+* must be context-free, including *L*.
  3. True;
  4. False;
  5. *L* = {*ww* : *w* ∈ (*ab*)n : *n* ≥ 0} is a regular language. Through proof by construction of a DFSM:   
     

*Fig. 8.1: DFSM for   
L* = {*ww* : *w* ∈ (*ab*)n : *n* ≥ 0}

* 1. *L* = {*aibkcidk*: *i*, *k* ≥ 0} is not context-free.

Through proof by contradiction and the Pumping Theorem, assume *L* to be a context-free language.

Let *w = akbkckdk*.

Where can be written *w* = *uvxyz*, |*vxy*| ≤ *k,* |*vy*| ≠ ε **and** (*uvnxynz* ∈ *L* Ɐ*n* ≥ 0), we designate sections |*ak*|*bk*|*ck*|*dk* to be respectively 1 | 2 | 3 | 4.

If we pump section 1, the string *ak+nbkckdk* is not in *L.*